

**Federal State Autonomous Educational Institution of Higher Education "Moscow  
Institute of Physics and Technology  
(National Research University)"**

**APPROVED**

**Head of the Phystech School of  
Applied Mathematics and  
Informatics**

**A.M. Raygorodskiy**

**Work program of the course (training module)**

**course:** Web-graphs/Веб-графы  
**major:** Applied Mathematics and Informatics  
**specialization:** Computer Science/Информатика  
Phystech School of Applied Mathematics and Informatics  
Chair of Discrete Mathematics  
**term:** 4  
**qualification:** Bachelor

Semester, form of interim assessment: 8 (spring) - Grading test

Academic hours: 60 AH in total, including:

lectures: 30 AH.

seminars: 30 AH.

laboratory practical: 0 AH.

Independent work: 75 AH.

In total: 135 AH, credits in total: 3

Author of the program: A.M. Raygorodskiy, doctor of physics and mathematical sciences, associate professor, head of chair

The program was discussed at the Chair of Discrete Mathematics 05.03.2020

## Annotation

In the course, students study modern mathematical models that allow them to describe the formation of the Internet with varying degrees of adequacy. Also, what practical use are similar models, after completing the course they will be able to conduct:

- analysis of a real graph;
- counting the characteristics of a real graph;
- approximation of properties, finding typical and atypical features;
- checking theoretical estimates and probabilistic models;
- researching the network structure, writing non-trivial analysis tools in Python.

### 1. Study objective

#### Purpose of the course

- mastering the basic concepts of the theory of webgraphs.

#### Tasks of the course

- Mastering by students of basic knowledge (concepts, concepts, methods and models) in the field of web graphs;
- acquisition of theoretical knowledge and practical skills in the field of web graphs;
- providing advice and assistance to students in conducting their own theoretical research in the field of webgraphs.

### 2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of studied phenomena and processes, systematically analyze scientific problems, obtain new scientific outcomes	Pro.C-1.1 Locate, analyze, and summarize information on current research findings within the subject area
	Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model
	IIK-1.3 Apply theoretical and/or experimental research methods to a specific scientific task and interpret the obtained results

### 3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- Fundamental concepts, laws, theories of random hypergraphs;
- modern problems of the corresponding sections of random hypergraphs;
- concepts, axioms, methods of proofs and proofs of the main theorems in the sections included in the basic part of the cycle;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems of random hypergraphs.

be able to:

- Understand the task at hand;
- use your knowledge to solve fundamental and applied problems of random hypergraphs;
- evaluate the correctness of the problem setting;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and analyze them;
- independently see the consequences of the results obtained;
- accurately present mathematical knowledge in the area of complex calculations, orally and in writing.

master:

- Skills of mastering a large amount of information and solving problems (including complex ones);
- skills of independent work and mastering new disciplines;
- culture of formulation, analysis and solution of mathematical and applied problems requiring the use of mathematical approaches and methods of random hypergraphs for their solution;
- the subject language of complex calculations and the skills of competently describing the solution of problems and presenting the results.

#### 4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

##### 4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Local Gallai-Erdős theorems on the number of vertices	5	5		10
2	Generalizations of Turan's Problem for Graphs and Hypergraphs	5	5		12
3	Basic definitions and concepts	5	5		8
4	The simplest problems of extremal graph theory	5	5		14
5	Connectivity. Spanning tree.	5	5		15
6	Transversal in a graph and the independence number	5	5		16
AH in total		30	30		75
Exam preparation		0 AH.			
Total complexity		135 AH., credits in total 3			

##### 4.2. Content of the course (training module), structured by topics (sections)

Semester: 8 (Spring)

###### 1. Local Gallai-Erdős theorems on the number of vertices

Turan's problem. Motzkin-Strauss theorem. Generalizations for hypergraphs. Turanian-type problems for classes of graphs and hypergraphs from combinatorial geometry.

###### 2. Generalizations of Turan's Problem for Graphs and Hypergraphs

The extremal problem on graphs without cycles of length 4 and finite projective planes.

###### 3. Basic definitions and concepts

Graphic sequences. Algorithm of determination, graphic sequences and the Gallai-Erdős theorem.

###### 4. The simplest problems of extremal graph theory

Independence number and click number. Ramsey's theorem (reminder) and the  $(p, q)$  property.

Graph independence function. Bipartition criterion and independence function. Ramsey-type problems for classes of graphs and hypergraphs from combinatorial geometry.

5. Connectivity. Spanning tree.

Various spanning tree problems.

6. Transversal in a graph and the independence number

Edge graphs and Gallai's maximum paracombination theorem.

## **5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)**

Standard classroom.

## **6. List of the main and additional literature, that is necessary for the course (training module) mastering**

### **Main literature**

1. Экстремальные задачи теории графов и Интернет [Текст] : [учеб. пособие для вузов] / А. М. Райгородский .— Долгопрудный : Интеллект, 2012 .— 104 с.
2. Модели Интернета : учебное пособие / А. М. Райгородский .— 2-е изд. — Долгопрудный : Интеллект, 2019 .— 64 с. - ISBN 978-5-91559-260-4 .— Электрон. версия печ. публикации .— URL: <http://books.mipt.ru/book/301332> (дата обращения: 21.12.2020). - Полный текст (Режим доступа : из сети МФТИ / Удаленный доступ).

### **Additional literature**

1. Графы. Алгоритмы на языке C [Текст] : учеб. пособие для студентов 1 курса МФТИ / В. В. Прут ; М-во образования и науки РФ, Моск. физ.-техн. ин-т (гос. ун-т) .— М. : МФТИ, 2017 .— 213 с. + pdf-версия. - Библиогр.: с. 207-209. - 200 экз. - ISBN 978-5-7417-0633-6 .— Полный текст (Доступ из сети МФТИ).

## **7. List of web resources that are necessary for the course (training module) mastering**

<http://dm.fizteh.ru/>

## **8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)**

Multimedia technologies can be used in lectures and practical exercises, including presentations.

## **9. Guidelines for students to master the course**

1. It is recommended to successfully pass test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

**Assessment funds for course (training module)**

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**Author:** A.M. Raygorodskiy, doctor of physics and mathematical sciences, associate professor, head of chair

## 1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of studied phenomena and processes, systematically analyze scientific problems, obtain new scientific outcomes	Pro.C-1.1 Locate, analyze, and summarize information on current research findings within the subject area
	Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model
	IIK-1.3 Apply theoretical and/or experimental research methods to a specific scientific task and interpret the obtained results

## 2. Competency assessment indicators

As a result of studying the course the student should:

### know:

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- Understand the task at hand;
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- Skills of mastering a large amount of information and solving problems (including complex ones);
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## 3. List of typical control tasks used to evaluate knowledge and skills

In order to control the mastering of educational material by students, an oral survey is conducted at the beginning of the lesson on the topic of the previous lesson.

## 4. Evaluation criteria

1. Basic definitions and concepts.
2. Graphic sequences. Algorithm of determination, graphic sequences and the Gallai-Erdős theorem.
3. Connectivity. Spanning tree. Various spanning tree problems.
4. The simplest problems of extremal graph theory.
5. Independence number and clique number. Ramsey's theorem (reminder) and the  $(p, q)$  property. Graph independence function. Bipartition criterion and independence function. Ramsey-type problems for classes of graphs and hypergraphs from combinatorial geometry.
6. Transversal in the graph and the number of independence. Edge graphs and Gallai's maximum paracombination theorem.

7. Local Gallai-Erdős theorems on the number of vertices and Bollobás's theorem on the number of edges, guaranteeing the existence of a  $k$ -transversal. Generalizations of these theorems for hypergraphs.
8. The task of Turán. Motzkin-Strauss theorem. Generalizations for hypergraphs. Turanian-type problems for classes of graphs and hypergraphs from combinatorial geometry.
9. Generalizations of Turán's problem for graphs and hypergraphs.
10. The extremal problem on graphs without cycles of length 4 and finite projective planes.
11. Schenkov graph capacity and Lovász's theorem on the capacity of a cycle of length 5.

- the mark "excellent (10)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to confidently apply them in practice in solving specific problems, free and correct justification of the decisions

- the mark "excellent (9)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, free and correct justification of decisions;

- the mark "excellent (8)" is given to a student who has shown a comprehensive systematized, deep knowledge of the curriculum of the discipline and the ability to apply them in practice when solving specific problems, and the correct justification of the decisions made;

- the mark "good (7)" is given to a student if he firmly knows the material, expresses it competently and to the point, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (6)" is given to the student if he knows the material, presents it competently and in essence, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (5)" is given to a student if he knows the material, and essentially expounds it, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "satisfactory (4)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, a violation of the logical sequence in the presentation of the program material, but at the same time he owns the main sections of the curriculum necessary for further education and can apply the obtained knowledge by model in a standard situation;

- the mark "satisfactory (3)" is given to a student who has shown a fragmentary, scattered nature of knowledge, insufficiently correct formulations of basic concepts, violation of the logical sequence in the presentation of program material, but at the same time he has fragmentary knowledge of the main sections of the curriculum necessary for further education and can apply the knowledge gained by the model in a standard situation;

- the mark "unsatisfactory (2)" is given to a student who does not know most of the main content of the curriculum of the discipline, makes gross mistakes in the formulation of the basic concepts of the discipline and does not know how to use the knowledge gained in solving typical practical problems;

- grade "unsatisfactory (1)" is given to a student who does not know the formulations of the basic concepts of the discipline.

## **5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience**

During the test, students can use the discipline program.